Control experiments for a network of signalized intersections using the '.Q' simulator

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Abstract:
The control of a network of signalized intersections is studied using a discrete-event simulator called 'point queue' (.Q). Vehicles arrive at entry links from outside the network in a continuous Poisson stream, independently make turns at intersections, and eventually leave from exit links. There is a separate queue at each intersection for each turn movement. The control at each intersection determines the amount of time that at each queue is served within each cycle. A vehicle arriving at an intersection joins the appropriate queue, waits there until it is served (its 'green light' is actuated), then travels over the downstream link and joins the next queue or leaves if it is an exit link. The performance of the control scheme is measured in terms of the length of each queue, the queue waiting time, or the travel time from entry to exit. Two sets of control policies are modeled and compared via .Q simulations for a fairly complex arterial network near the I-15 freeway in San Diego, CA. The first is 'fixed time (FT)' control which generates an open loop periodic sequence of green light actuations. The second is a feedback strategy called 'max pressure (MP)' in which the turn movement that is actuated is a function of the queue lengths adjacent to the intersection. The simulations confirm the theoretical property of MP, namely that it maximizes throughput, whereas FT does not. The simulation study provides more details concerning the queue length distribution and the behavior of MP as a function of how frequently it is invoked. These details are critical in evaluating the practicality of MP. The study shows that the .Q simulator is a versatile tool in the design of signal control.

Keywords: Traffic-responsive signal control, stabilizing policy, fixed-time control, max-pressure control, store and forward queueing model, discrete-event simulation, Monte Carlo simulation, network performance evaluation.

1. INTRODUCTION

A network of signalized intersections is modeled as a network of controlled queues, with one queue per movement or phase, like in a store-and-forward communication network. At any time, a control policy actuates a stage, i.e. a set of simultaneous movements, for a duration of time. The actuation of a movement causes the corresponding queue to be served. The service rate, called the saturation flow rate, is pre-specified and depends on the geometry of the intersection. When a non-empty queue is served vehicles move towards the downstream queue and join it after a pre-specified link travel time. Each link has a pre-specified finite storage limit depending on the link geometry. If the downstream link has reached its limit, the upstream queue is blocked even if it is actuated by the control.

The large literature on signal control policies is reviewed in Mirchandani and Head (2001); Papageorgiou et al. (2003); Osorio and Bierlaire (2008); Xie et al. (2012). Each study proposes an intuitively appealing policy, supported by an illustrative simulation, since mathematical analysis of a store-and-forward queuing network with blocking seems impossible. Two control schemes are compared in this paper in the context of the arterial network near the I-15 freeway in San Diego, CA, shown in Fig. 1. A fixed-time control policy or FT is an open loop periodic sequence of stage actuations. In max pressure control or MP the stage that is actuated is a function of the queue lengths adjacent to the intersection. Theoretical properties of FT and MP are derived in Varaiya (2013b,a). Practically more important properties are compared here via simulation.

The study network of Fig. 1 is a graph with nodes or intersections like 37612 at the top and directed links entering and leaving the nodes. Link 741 is an entry link, 742 is an exit link, and 737 is an internal link. A queue is associated with each incoming and outgoing link pair: thus $q(741, 737)$ is the number of vehicles at this intersection on link 741 waiting to make the turn $(741 \to 737)$. When the stage that includes this turn is actuated, vehicles will leave this queue at a rate given by the saturation flow rate $C(741, 737)$, provided that the downstream link 737 is not full. (If that link is full, the turn is blocked even if it is actuated.) Once a vehicle leaves $q(741, 737)$ it will travel along link 737. (In the study the travel time is a lognormal iid sequence.) When the vehicle arrives at the downstream intersection, namely 37610, it will join a queue for one of the four outgoing links, 733, 738, 2351, or 20471, with a pre-defined turn probability. (In this study all turn probabilities are taken equal.)

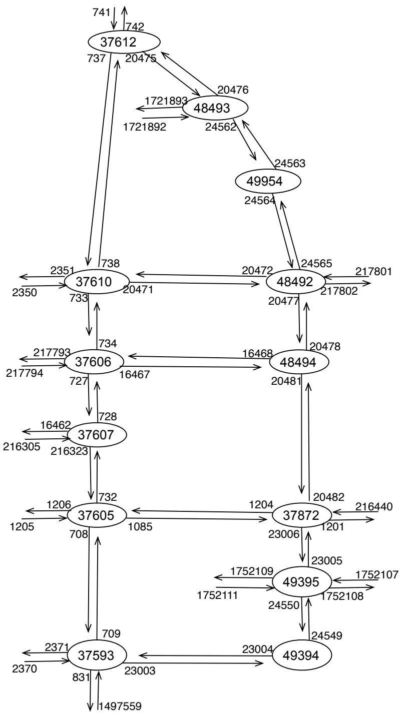

Fig. 1. Study network near I-15 San Diego, CA

Vehicles enter the network at entry links in a Poisson stream with specified demand rate. Suppose d is the vector of demand rates, with $d_l = 0$ if l is not an entry link. Let $R = \{R(l,m)\}$, with $R(l,m)$ equal to the probability that a vehicle on link l turns into link m. If the network is stable, the vector $f = \{f_l\}$ of average link flows must satisfy the conservation equation $f = R'f + d$; hence $f = [I - R']^{-1}d$ (R' is the transpose of R), and $R(l,m)f_l$ is the average rate of turns from link l to m. If a control policy can support demand d, it must actuate movement (l,m) at rate at least $R(l,m)f_l$. If this condition holds, the queues are stable, i.e. ($\mathbf{E}$ denotes expectation)

$$\sup_T T^{-1} \sum_{l,m} \sum_{t=1}^{T} \mathbf{E}q(l,m)(t) < \infty. \tag{1}$$

Stability is an essential requirement for a signal control policy. But beyond stability, one wants good performance in terms of the average queue size, or the average total travel time from entry to exit, or the average and variance of the travel time along important routes, etc. All these quantities require simulation.

2. STABILITY, FT, AND MP

At each intersection a control actuates a stage (a set of simultaneous turn movements). So a control is represented by a matrix U with $U(l,m) = 1$ or 0, accordingly as turn (l,m) is or is not actuated. Let $C = \{C(l,m) \geq 0\}$ be the matrix of pre-specified saturation flow rates (in units of vehicles per hour). Then $\{C(l,m)U(l,m)\}$ is the matrix of service rates at which queues at this intersection are served when control U is invoked. The control for the entire network at any given time consists in selecting a control $U(n)$ from the finite set of controls $\mathcal{U}(n)$ for each intersection n.

We now specify a FT control. We are given a cycle time T, a loss time $L < T$, and a fixed sequence of stages $U_1, \cdots, U_K$ from $\mathcal{U}(n)$ for intersection n. A FT control for intersection n is specified by a vector $\lambda = (\lambda_1, \cdots, \lambda_K)$ such that $\lambda_i \geq 0$ and $\sum_i \lambda_i = 1 - L/T$. λ specifies the periodic control: invoke U_1 for duration $\lambda_1 T, \cdots,$ invoke U_K for duration $\lambda_K T$; then repeat. L is the time within each cycle that is 'lost' because of the safety-required 'all red' signal between stage switches (typically 3s) and for pedestrian crossing. (At a US intersection, K is 4-8, T is 60-120s, L is 10-20s.) If FT control λ has been selected at intersection n, the resulting matrix of service rates is $\{\sum_i \lambda_i C(l,m)U_i(l,m)\}$.

If d is the average demand rate vector, R is the turn probability matrix, the average link flow vector is $f = [I - R']^{-1}d$ and $R(l,m)f_l$ is the average rate of turns needed to meet the demand. Hence if the FT control λ at an intersection is to support this demand, it must satisfy the inequalities

$$\begin{gathered} \sum \lambda_i C(l,m)U_i(l,m) > R(l,m)f_l, \text{ all } l,m \\ \sum \lambda_i = 1 - L/T; \ \lambda_i \geq 0, \text{ all } i. \end{gathered} \tag{2}$$

Varaiya (2013a) shows that (2) is sufficient for stability. Observe that to design a stable FT control one must know d and R, in addition to the physical saturation flows C. The set of stages $\mathcal{U}(n)$, which determines whether and which turns are allowed, is designed by traffic planners. The number K of stages and the stage sequence can be included in condition (2). If (2) can be satisfied, infinitely many λ are feasible. One criterion to select the 'optimum' λ is to maximize the minimum 'excess capacity',

$$\min_{l,m} \sum_i \lambda_i C(l,m)U_i(l,m) - R(l,m)f_l.$$

Both d and R change over the time of day and day of week, and when there are unusual events. An FT λ that satisfies (2) for one demand may not satisfy it for another demand. One advantage of MP is that it automatically adapts to changes in the demand vector d.

MP is specified as follows. T, L are as before. The cycle is divided into a number of equal periods (between 2 and 10 in the simulations below). MP selects the stage to be actuated in each period, depending upon the queue length measurements made just before the start of the period. Suppose at intersection n the queue measurements are $q(t) = \{q(l,m)(t)\}$. Compute the 'pressure' $\pi(U, q(t))$ exerted by each control $U \in \mathcal{U}(n)$:

$$\pi(U, q(t)) = \sum_{l,m} C(l,m)U(l,m)W(l,m) \tag{3}$$

$$W(l,m) = q(l,m) - \sum_p R(m,p)q(m,p)]. \tag{4}$$

The MP policy selects the control with maximum pressure at state $q(t)$,

$$U^*(q(t)) = \arg\max\{\pi(U, q(t)) \mid U \in \mathcal{U}(n)\}. \tag{5}$$

$W(l,m)$ is the upstream queue minus the (average) downstream queue for movement (l,m); it is called the weight of the phase (l,m) in Varaiya (2013a). So MP selects the stage that maximizes the instantaneous rate at which W decreases. The stage U^* is actuated until the end of the period, when the queue is measured again and a new MP is computed. Observe that U^* does not depend on the demand rate d, but it does depend on the turn probabilities R.

The main theoretical result in Varaiya (2013a,b) is that MP will stabilize the network if there exists a feasible solution to (2). Thus if the demand changes over time, MP may maintain stability even when there is no single stabilizing FT. It is also proved that if R is estimated (perhaps by counting turns) in a consistent manner, one may replace R in (4) by its estimate and the stability property is maintained. In this sense MP is adaptive. Another property proved in Varaiya (2013a) but not explicitly called out is that the average queue size $\mathbf{E}\sum q(l,m)(t)$ is proportional to $1/P$, where P is the number of times within a cycle that a new MP control is selected. However, there is no quantitative comparison between the performance of FT vs MP. That comparison is carried out below using the .Q simulator.

3. .Q SIMULATOR

A discrete event simulator like .Q is specified by a set of events and a procedure for treating each event. The occurrence of an event modifies the state according to the associated procedure, and it may also trigger future events. The state of .Q is the vector of queue lengths, together with some memory needed to specify the control, and some memory required to trace the movement of a vehicle from one intersection to the next. .Q has two principal classes of events.

1. Events initiated by vehicles:
   (a) Vehicle Appears At An Entry Link: A new vehicle appears at the network;this will also trigger the next arrival.
   (b) Vehicle Arrives At A queue: A vehicle joins a queue.
   (c) Vehicle Ends Its Hold Time: The time, which depends on the saturation rate, that a vehicle at the head of a queue is held there before it can leave.
   (d) Vehicle Departs A Queue: A vehicle leaves its queue and heads towards its next destination intersection.
2. Events initiated by the intersection control:
   (a) Decision For the Next Traffic Control: The selection of the next stage to actuate at the end of the duration of the currently actuated stage. For FT this is pre-defined; for MP this will depend on the calculation (5).
   (b) New Network Control: When the actuation follows the previously selected control.

The .Q simulator records the system trajectory, i.e., the time-stamped sequence of events and corresponding states. Many trajectories may be recorded for a stochastic simulation. Performance of the controller is obtained by appropriately processing the recorded trajectories.

Following data are needed for a .Q simulation. (1) Network: graph; length, storage capacity and travel time of each link; saturation flow rate of each turn; control stages $\mathcal{U}(n)$ for each intersection. (2) Control: FT or MP; .Q also has procedures to simulate actuated control. (3) Demand: average demand rates, turn probabilities; .Q can also use demand specified via origin-destination-path flows. In the simulations below, the time step is 0.1s and each simulation lasts 10,800s or 3 hours.

.Q is sometimes called a mesoscopic in contrast with microscopic and macroscopic traffic simulators. A microscopic simulator models the movement of individual vehicles each using a simple differential equation that emulates car-following and lane-changing behavior of drivers. A microscopic simulation model has hundreds of differential equations (vehicles) each with its own driver behavior parameters, so the model is impossible to calibrate. The usefulness of commercial microsimulation packages is their impressive '3D' visualization of the results, which lends verisimilitude but no statistical guarantee. Of course, microscopic simulators are slow, so running stochastic simulations is time-consuming.

Macroscopic simulators typically model traffic flow as a fluid, often based on the cell transmission model (CTM), see Lo (2001). They are straightforward to calibrate, but it is quite difficult to model turns, shared lanes and queues. Feedback controllers in practice rely on sensors that are actuated by the passage of vehicles. It is a challenge to model such sensors in a CTM model of a fluid whose state is density (vehicles/km) and use it to predict the performance of (say) a fully actuated controller. Random (Poisson) demands and exponential service are also hard to model.

A mesoscopic simulator, like .Q, can incorporate the advantages of both. It requires the same model parameters as macroscopic models (network, demand, control), and they do model individual vehicles, but it doesn't model lane changing or car-following. It is easier to calibrate than a macroscopic model because it involves vehicle events that are measured by standard detectors. It is much faster than microsimulations. (A 10,800s .Q simulation of the 13-node, 150-queue study network takes under 1s on a laptop computer.) Since it models individual vehicles, one can model sensors that detect their passage and hence actuated control.

4. FT AND MP, BASIC PROPERTIES

The demand d for the network of Fig. 1 was obtained from an Aimsun microsimulation of the network. (As noted before, turns are taken to be equiprobable. An FT control $\lambda = \{\lambda(n)\}$ (n is intersection index) is obtained by solving the LP problem obtained by adding the criterion of maximizing the excess capacity to the inequalities (2). It turns out that (2) is feasible. Fig. 2 plots the sum of all queues $\sum_{l,m} q(l,m)(t)$ every 0.1s for 10,800s or 3 hours. (An argument in Varaiya (2013a) can be used to prove a kind of ergodicity result, namely that the time average of this queue-sum converges to its statistical average.) The plot confirms the prediction of stability of this FT control.

Figure 3 plots the evolution of the sum of the queues, for the same demand d under MP control (5) when the number of the control decisions taken per cycle, is varied from two to ten decisions. This verifies the property predicted in Varaiya (2013a) that the queue size will decrease as the MP update is taken more frequently.

A comparison of the plots in Figs.2 and 3 shows that the queues under MP with 4 decisions/cycle (which is also the number of stages per cycle under FT) leads to queue sums

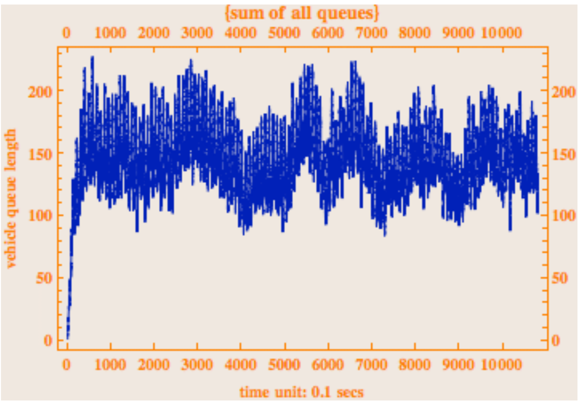

Fig. 2. Sum of queues with demand d and FT control λ

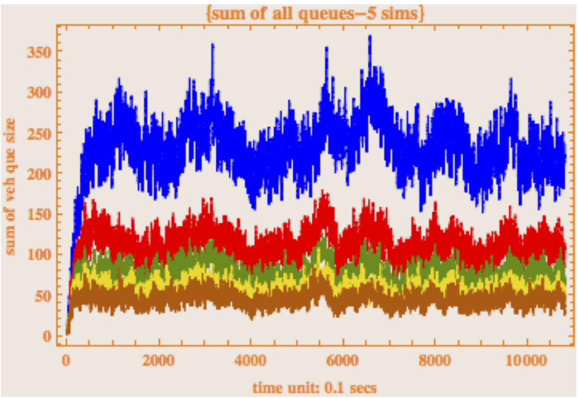

Fig. 3. Sum of queues with demand d and MP control with 2 (blue), 4 (red), 6 (green), 8 (yellow), and 10 (brown) decisions/cycle

that are smaller on average and have smaller variability. This property is not predicted in the theory but it is (presumably) a consequence of the adaptability of MP.

It is impractical to reduce queues simply by invoking MP more frequently within each cycle, because changing a stage incurs a loss. A more practical approach, which we call MP-pract, is to evaluate MP frequently, but implement a change only if it leads to a significantly larger pressure:

$$\max_{U} \pi(U, q(t)) - \pi(U^*, q(t)) > \eta. \quad (6)$$

Here U^* is the previously selected MP stage, and the threshold $\eta > 0$ should be chosen to prevent excessive stage switching. Figure 4 plots the evolution of the sum of queues under MP-pract with evaluations two, four, six, eight and ten times per period, and η small.

Figs. 3 and 4 show that the performance of MP-pract is comparable to that of MP but requires much fewer stage changes, as Table 1 reveals. The Table compares for intersection 37593 the total number of MP evaluations vs MP-pract implementations. Interestingly, the number 899 of MP-pract implementations for 8 decisions/cycle is smaller than the number 938 of MP implementations for 4 decisions/cycle, which is also the number of stage switches using FT. Thus MP-pract appears to be a practically sound implementation of MP.

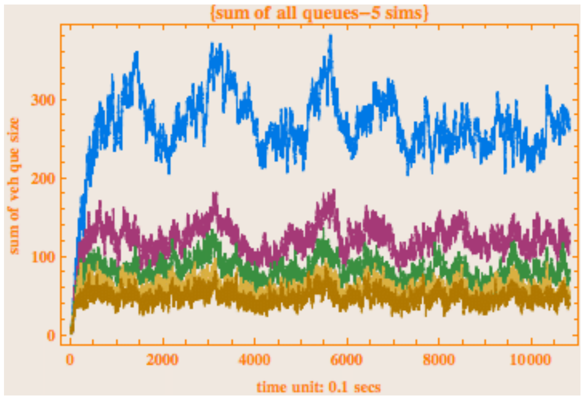

Fig. 4. Sum of network queues with demand d and MP-pract 2,4,6,8,10 decisions/cycle following (6).

Nb MP Decisions Per Period	**Total Nb Evaluations**	**Total Nb implementations**
2	466	224
4	938	455
6	1403	680
8	1866	899
10	2243	1026

Table 1. Number of MP evaluations vs number of MP-pract implementations for intersection 37593.

5. NETWORK BEHAVIOUR UNDER DEMAND VARIATION

We consider a time-varying demand equal to d_1 from time 0 to 10,800 s and d_2 from time 10,801 to 21,600. The only difference between d_1 and d_2 is the demand at the entry link 2370 at intersection 37593, at the bottom of Fig.1. FT control λ_1 supports d_1, λ_2 supports d_2, but no FT control supports both d_1 and d_2. We consider $q(2370, 709)$ on link 2370.

Fig. 5 and 6 depict the evolution of $q(2370, 709)$ when FT λ_1 is employed for the entire 6-hour period. As can be seen, the system is stable for the first 3 hours (with demand d_1) but then becomes unstable with demand d_2. However, FT λ_2 supports demand d_2 as seen in Fig. 7, but λ_2 does not support d_1 (not shown).

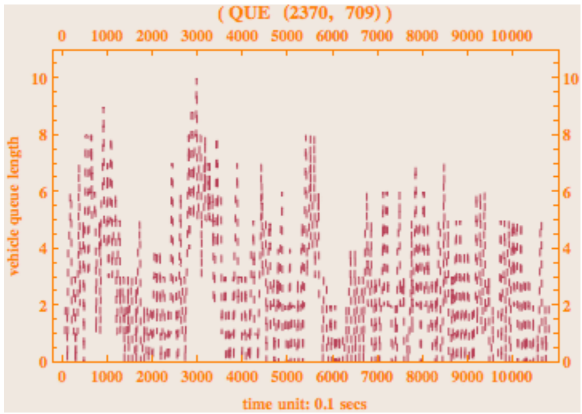

Fig. 5. $q(2370, 709)$ - FT control (d_1, λ_1).

Figs. 8,9 show the evolution of $q(2370, 709)$ when demand level varies from d_1 to d_2 under MP control. Evidently,

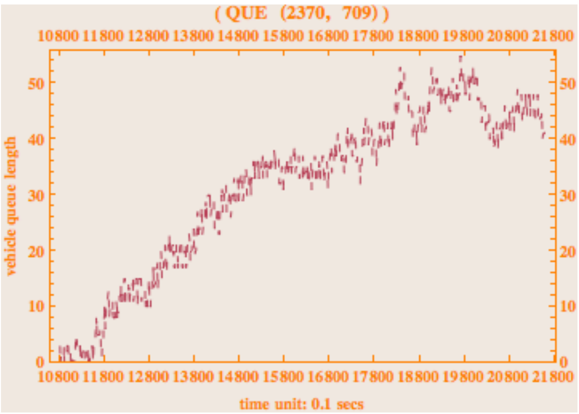

Fig. 6. $q(2370, 709)$ - FT control (d_2, λ_1).

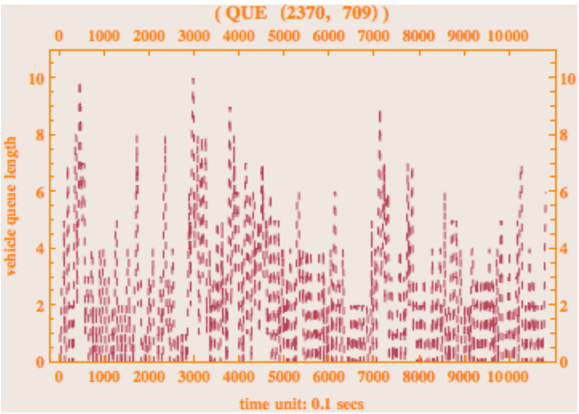

Fig. 7. $q(2370, 709)$ - FT control (d_2, λ_2).

MP adapts to the change in demand, in sharp contrast with FT.

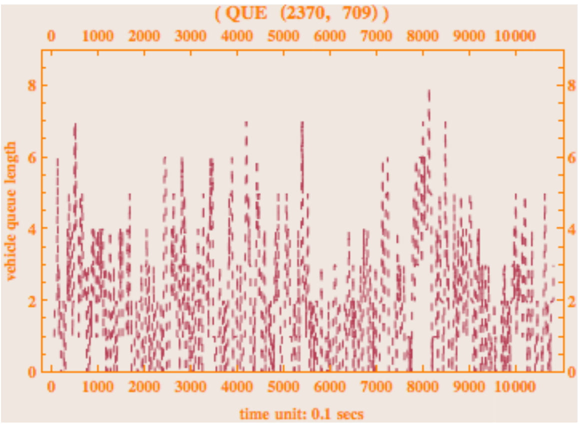

Fig. 8. $q(2370, 709)$ - MP control and demand d_1.

6. TRAVEL TIME

Travel time is an important performance metric of signal control strategies.

Table 2 compares the mean travel time (MTT) and the number of completed trips for three different stabilising policies with the same demand. The trips are those that originate at node 37593 and leave from one of the exit links. As expected, because all controls are stabilizing, the same number of vehicles complete the trips. However, the mean travel times for FT are much larger than for MP and MP-pract. Further, the travel times for MP and MP-pract are similar.

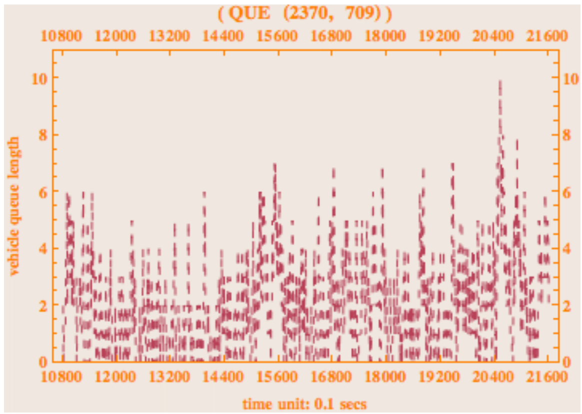

Fig. 9. $q(2370, 709)$ - MP control and demand d_2 continuation of demand d_1.

(Exit, Entry) Link	FT control Number Veh & MTT		MP 8 ctrls per cycle Number Veh & MTT		MP Practical 8 ctrls per cycle Number Veh & MTT	
(2370, 217793)	218.6	27	141.3	27	149.1	27
(2370, 742)	300	12	246.9	12	255.2	12
(2370, 16462)	207.5	46	136.7	47	128.50	46
(2370, 217802)	216.8	19	187.3	19	210.6	20
(1497559, 742)	340.5	9	243.8	9	274.5	9
(11497559, 1721893)	298.8	10	228.3	10	243.9	10
(1497559, 217802)	219.4	11	180.1	11	214.5	11

Table 2. MTT in sec and number of completed trips with FT, MP, MP-Pract.

The trip travel time is the sum of the link travel times and the queue sojourn times. Since link travel times do not depend on the control policy, the difference in trip travel times is due to differences in queuing delay. Table 3 shows the number of vehicles and the mean sojourn time in each queue of the intersection node 37593 with 2 to 10 MP decisions per cycle. The smaller queuing delay under MP with more frequent updates per cycle conforms to the correspondingly lower queue size shown in Fig. 3. As seen in Table 4, the delay incurred by FT is larger than by MP with 4 or more MP decisions per cycle,.

Finally, Tables 5 and 6 show how the MTT for trips originating in entry link 2370 to selected exit links changes under MP with 2 to 10 decisions per cycle. Once again if we recall that the performance of MP with 8 or 10 decisions per cycle is very similar to that of MP-pract with 4 implementations per cycle, we see that MP-pract offers a very satisfactory control performance.

7. CONCLUSION

The paper illustrates the use of the simulator .Q to evaluate the performance of the control of a network of signalized intersections. The control of such a network actuates a stage (i.e. a set of simultaneous turn movements) at each

(Input, Output) Link	Nb Stat. Veh.	Mean Time Spent in Que, (secs) by Veh. MP 2,4,6,8, 10 updates per cycle				
(23004, 831)	424	67.1	30.7	21.5	15.4	12.7
(23004, 2371)	430	55.5	28.2	20.4	16.5	13.1
(2370, 709)	336	131.7	73.4	40.8	31.4	21.7
(2370, 23003)	298	97.1	48.3	32.8	28	19.9
(1497559, 2371)	234	73.9	31.2	18.1	15.1	11
(1497559, 709)	280	109.4	58	39.6	27.6	20.1
(708, 23003)	863	26.2	15.4	10.6	8.8	7.9
(708, 831)	806	32.4	18.9	13.1	11.3	9.8

Table 3. Mean sojourn time in queues at node 37593 with 2, 4, 6, 8, & 10 MP decisions/cycle.

(Input, Output) Link	Nb Stationned Veh.	Mean Time Spent in Que, (secs) by Veh. FT stabilizing
(23004, 831)	421	37.5
(23004, 2371)	433	23.7
(2370, 709)	336	41.5
(2370, 23003)	298	35.3
(1497559, 2371)	234	30.1
(1497559, 709)	280	23.5
(708, 23003)	864	30.8
(708, 831)	809	35.1

Table 4. Mean sojourn time in queues at node 37593 with 'optimized' FT control.

(Entry,Exit) Link	Nb Veh. Served & MTT (secs) 2 MP updates per cycle		Nb Veh. Served & MTT (secs) 4 MP updates per cycle		Nb Veh. Served & MTT (secs) 6 MP updates per cycle	
(2370, 1752108)	132	176.6	137	106	137	89.1
(2370, 831)	355	25.9	355	15.6	355	11.7
(2370, 1201)	56	276.3	56	170.3	56	120
(2370, 217793)	26	363.8	27	207.4	27	164.5
(2370, 2351)	14	439.1	14	332.4	14	234.2
(2370, 742)	12	608.4	12	371.9	12	284.6
(2370, 1721893)	11	550.2	11	360.6	11	301.9
(2370, 217802)	19	441.7	19	285.9	19	216.8
(2370, 1752109)	114	188.2	115	108.2	115	80.3
(2370, 16462)	45	406.2	46	232.9	47	165.1
(2370, 1206)	149	328.1	154	196.1	152	113.6

Table 5. MTT from entry link 2370 at node 37593 under 2, 4, 6 MP decisions/cycle.

time and in every intersection. There is a large literature on the design of such controls. Invariably, the performance of a control scheme is carried out via simulation. This paper compares two families of controllers in the context of a signalized arterial network with 13 nodes, 50 links and 150 queues.

.Q models the network as a store-and-forward queuing network. It is a mesoscopic simulator: it models the movement of individual vehicles, but without lane-changing. It is a discrete-event simulator that retains only the essential

(Entry,Exit) Link	Nb Veh. Served & MTT (secs) 8 MP updates per cycle		Nb Veh. Served & MTT (secs) 10 MP updates per cycle	
(2370, 1752108)	138	73.3	137	63.9
(2370, 831)	355	10.2	355	9.1
(2370, 1201)	56	106.1	56	88.1
(2370, 217793)	27	141.3	27	121.3
(2370, 2351)	14	192	14	167.5
(2370, 742)	12	246.9	12	188.7
(2370, 1721893)	11	253.6	11	240.8
(2370, 217802)	19	187.3	19	145
(2370, 1752109)	115	75.8	116	62.9
(2370, 16462)	47	136.7	46	104.1
(2370, 1206)	153	92.6	155	76.5

Table 6. MTT from entry Link 2370 at node 37593 with 8, 10 MP decisions/cycle.

events needed to describe a vehicle's trajectory (entering, joining and leaving a queue, making a turn, departing the network) and control decisions (which and when a stage is selected and actuated). .Q is easy to use, fast and versatile. The last property is illustrated by the two control schemes that are studied: FT or fixed-time control and MP or max-pressure control. Stability of FT and MP has already been theoretically investigated. .Q is used to compare their performance in simulations.

The simulation study confirms the theoretical predictions about stability and instability. It confirms that MP is adaptive: it preserves stability in the face of demand variation. More interestingly it shows that queues decrease as the MP control is invoked more frequently. Of practically greater importance is the finding that with MP-pract the number of stage switches can be limited without sacrificing the gain from more frequent switching.

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