Variants of max pressure control for a network of signalized intersections

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***Abstract*—*Max-Pressure* (MP) policies applied to a Store and Forward network model, are based on queue measurements. The principal distinction of the MP control employed in this paper from those proposed in previous studies such as [1], [2], [3], etc. consists in the *decentralised* character of this new version accompanied by stability guarantees. At each intersection the choice of the movements to actuate depends on the size of the adjacent queues to the intersection. Moreover, no knowledge of the external arrivals is required but only turning ratios. On the other hand, as presented in [4], whenever there exists any stabilising policy keeping all network queues bounded, the *Max-Pressure* also stabilises the system. Within this paper, *Pre-timed* controllers, periodically distributing the same actuation duration to each set of permitted movements, are compared with versions of MP control, studied in [4], [5], under fixed or fluctuating demand levels. Discrete event simulations evaluate the network performance for each policy in terms of travel times, queue size and delays.**

***Keywords*-Traffic-response signal control; stabilising policy; fixed-time and max-pressure controllers; store and forward queueing model; discrete-event simulation; network performance evaluation**

I. CASE STUDY

A network of two signalised intersections is being considered, as shown in Figure 1.

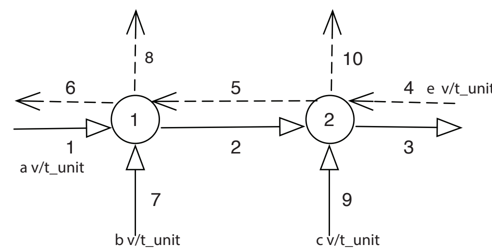

Figure 1. Network

A *phase* (l, m) is a movement from link l to link m.

If P_{n_i} denotes the set of permitted phases of intersection $n_i, i = 1, 2$ then $P_{n_1} = \{(1, 2), (5, 6), (7, 2), (7, 6), (7, 8)\}$ and $P_{n_2} = \{(2, 3), (4, 5), (9, 3), (9, 5), (9, 10)\}$.[1]

$s(l, m)$ represents the saturation flow of phase (l, m), measured in vehicles per time period.[2] The saturation flow for every allowed movement is taken equal to 1188 vehicles per hour.

$\gamma(l, m)$ is the turning ratio of phase (l, m), expressed as the probability of a vehicle to choose as destination link m when joining link l. During the present study, all permitted movements originated in the same link, are equiprobable.

A. Traffic regulation: Admissible network control matrices

The simultaneously compatible movements of each intersection are represented by a binary matrix, the *intersection control matrix*, of which the (i, j) entry equals one if the corresponding phase is actuated, zero otherwise. If $U^i_{n_j}$ denotes the i-th *intersection control matrix* of node $n_j, j = 1, 2$ then there exist two intersection control matrices for each node of network 1,

$U^1_{n_1} : \{(1, 2), (5, 6)\}, U^2_{n_1} : \{(7, 6), (7, 8)\}$
$U^1_{n_2} : \{(2, 3), (4, 5)\}, U^2_{n_2} : \{(9, 10), (9, 5)\}.$

A *stage* or *network control matrix* is a block-diagonal matrix with matrices $U^i_{n_j}, j = 1, 2$ along the diagonal. If $\mathcal{U}$ denotes the set of admissible *network control matrices*, then $\mathcal{U} = \{U^1, U^2, U^3, U^4\}$, with

$$U^1 = \begin{bmatrix} U^1_{n_1} & 0 \\ 0 & U^1_{n_2} \end{bmatrix}; U^2 = \begin{bmatrix} U^1_{n_1} & 0 \\ 0 & U^2_{n_2} \end{bmatrix};$$

$$U^3 = \begin{bmatrix} U^2_{n_1} & 0 \\ 0 & U^1_{n_2} \end{bmatrix}; U^4 = \begin{bmatrix} U^2_{n_1} & 0 \\ 0 & U^2_{n_2} \end{bmatrix}.$$

B. Problem Statement

The optimisation horizon is divided into intervals or cycles of fixed width, each one comprising of T periods. Within each cycle, there exist $T - L$ available planning periods where $L < T$ represents the idle time corresponding to pedestrian movements, amber lights, etc. Let q be the array of which the (i, j) entry is the length of queue related to phase (i, j). The system state at time $t, X(t)$ is defined by $X(t) = q(t)$. A control *stabilises* the network, if the mean queue length is bounded.

Within each cycle *stage* $u(t) = U, U \in \mathcal{U}$ and $\lambda_{u(t)}$ cycle proportion have to be decided such that:

- $u(t)$ *stabilises* $X(t)$
- if $\tilde{c}(l, m)$ denotes the service rate of phase (l, m) and f_l represents the vehicle flow in link l, then the following

[1] Right turns are not submitted to any control, they simply contribute to vehicle arrivals at internal or exit network links. U-turns are not permitted movements during this study.

[2] The saturation flow of a non-permitted movement is considered equal to zero.

stability condition has to be verified,

$$\tilde{c}(l,m) > f_l \gamma(l,m), \forall (l,m) : s(l,m) > 0 \qquad (1)$$

- $\sum_{u \in \mathcal{U}} \lambda_u T + L \leq T.$

C. Modelling the network demand and travel times

Arrivals associated with each entry link are discrete in time and characterised by Poisson processes of which their parameters determine the demand intensity (per time unit) and may vary within the time to represent fluctuations of the demand in volume and geometry. Two demand vectors are considered, $d_1 = (0.015, 0.01, 0.005, 0.03)$ and $d_2 = (0.001, 0.01, 0.03, 0.01)$, each one is related to a distinct demand level.[3]

Stochastic travel times are associated with each entry or internal link, following a shifted lognormal distribution with a mean value of 30 seconds.[4]

II. Pre-timed network control

A *fixed-time* control is a periodic sequence, $\{\lambda_U,\ U \in \mathcal{U}\}$, actuating each *stage* $u(t) = U^i,\ U^i \in \mathcal{U}$ *for a fixed duration* $\lambda_{U^i} T$ *within every cycle of* T *periods.*

In what follows, the behaviour of network 1 is analysed when pre-timed *control rules the network.*

Stability condition (1) and demand d_1 *determine vector* $\lambda^1 = (0.57, 0.005, 0.15, 0.115)$ *such that* $\{\lambda^1 T,\ \mathcal{U}\}$ *stabilises network 1 under demand* d_1.

Experiments are performed according to a "micro" simulation functioning associated with stochastic travel times. A cycle duration T *of* 62 *seconds is considered of which* 10 *seconds correspond to the red clearance* L*, the time unit* t_{unit} *is taken equal to* 0.1 *seconds. The simulation duration is of* 8 *hours, (*28,800 *seconds, real time).*

Figure 2, illustrates the evolution of the sum of the network queues when fixed time *control* λ^1 *is applied to network 1 under demand* d_1*. As one may observe,* pre-timed *control* λ^1 *stabilises the network.*

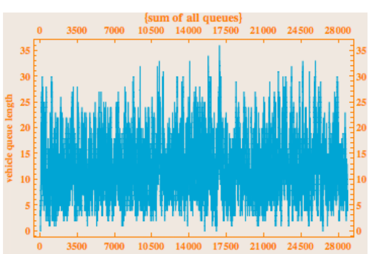

Figure 2. Sum of Vehicle Queues - (d_1, λ^1) - FT control

[3]Each coordinate of vector $d_i, i = 1, 2$ corresponds to values (a,e,b,c), demand parameters of entry links $1, 4, 7, 9$ respectively, (vehicles per time unit), see also Figure 1.

[4]Furthermore, deterministic times can be considered, according to the needs of the study.

However, as Figure 3 depicts, when the demand intensity varies, from d_1 *to* d_2*, the same* pre-timed *control* λ^1 *cannot maintain the system stable.*

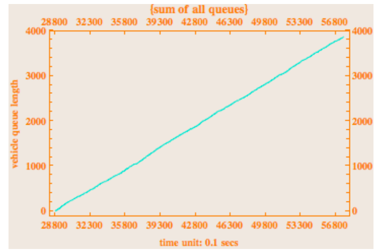

Figure 3. Sum of Vehicle Queues - (d_2, λ^1) - FT control

III. Max-Pressure control policy

The pressure $w(q(t), U)$ *exerted by* stage $U \in \mathcal{U}$*, is defined by*

$$w(q(t), U) = \sum_{(l,m)} \varsigma(l,m)(t) S \circ U(l,m)(t)^5 \qquad (2)$$

$$\varsigma(l,m)(t) = \begin{cases} q_{(l,m)}(t) - \sum_{p \in \mathcal{O}(m)} \gamma_{(m,p)} q_{(m,p)}(t), & \text{if } q_{(l,m)}(t) > 0, \\ 0, & \text{otherwise}. \end{cases}$$

At time t*,* Max-Pressure *control, selects to actuate the* stage *exerting the higher pressure to the network,*

$$U^*(q)(t) = \textit{argmax}\{w(q(t), U), U \in \mathcal{U}\}, \text{MP stage} \qquad (3)$$

A. Scheduling Max-Pressure controller

In the following analysis, within every cycle a max-pressure *stage is selected every* $(T - L)/4$ *time periods,* $\{u^i(t) = U^*(q)(t), \mu^i = (T - L)/4, i = 1, \ldots, 4\}$.

Figure 4 illustrates the sum of the network queues when MP control rules the network of demand d_1.

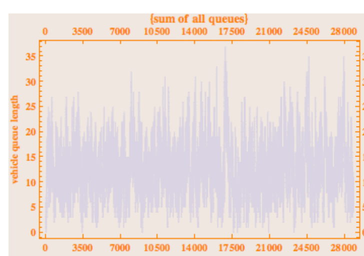

Figure 4. Sum of Vehicle Queues - $(d_1, \{\mu^i\}_{i=1}^4)$ - MP control

Figure 5 represents the sum of the network queues when demand level varies from d_1 *to* d_2*. In sharp contrast to* pre-timed *control,* max-pressure *stages stabilise the system under fluctuating demand level.*

[5]S is the matrix with the saturation flows of each phase, $q_{(i,j)}(t)$ is the state of the associated queue with phase (i, j) at time t, $\mathcal{O}(m)$ is the set of input links to link m.

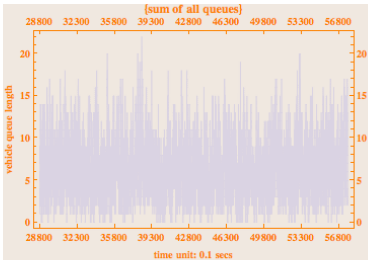

Figure 5. Sum of Vehicle Queues - $(d_2, \mu^i), i = 1, \ldots, 4$ - MP control

B. Max-Pressure with boundary control, (MP-BC)

An alternative of MP controller is now presented, according to which the number of vehicles directed towards a phase is also considered in the choice of the MP stage. More precisely, for any desired phase, whenever the corresponding queue reaches a maximum size, all movements towards the related link to the considered phase are prohibited. Moreover, in order to prevent a waste of green time, it is also examined whether this actuation duration can be allocated to a different phase. Hence, a Max-Pressure *stage* $U^*(q)$ *is selected according to equation (3).*

Whenever $q(j,k)$ oversized *and* $U^*(i,j) = 1$ *then*

- $U^*(i,j) = 0, \forall i \in \mathcal{I}(j), \forall (j,k) : j \in \mathcal{I}(k)$[6]
- *a* Max-Pressure *intersection stage is selected, for the head node of link* i.

Demand vector $d_3 = (0.045, 0.01, 0.005, 0.03)$ *is employed during this analysis.*[7] *The surveyed phase is* $(2,3)$ *and the desired size of queue* $q_{(2,3)}$ *is taken equal to* 20.

Figure 6 illustrates the evolution of queue $q_{(2,3)}$ *during an implementation of* 3600 *seconds, when* MP *control is applied to network 1 of demand* d_3. *The slight increase of the queue length (*25 *vehicles) is due to the vehicles departed before queue* $q_{(2,3)}$ *reaches the maximum permitted size.*

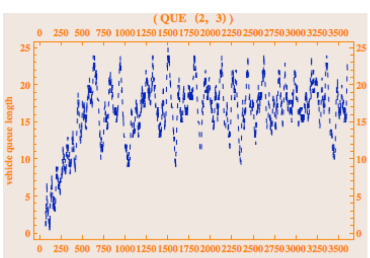

Figure 6. Queue $(2,3)$ - $(d_3, \{\mu^i\}_{i=1}^4)$ - MP control

Figure 7 illustrates the evolution of queue $q_{(7,6)}$ *when the actuation duration of phase* $(2,3)$ *is redistributed to other phases, every time queue* $q_{(2,3)}$ *is oversized.*

[6] $\mathcal{I}(k)$ is the set of input links to link k

[7] the corresponding demand of entry link 1 is now increased comparatively to demands d_1, d_2

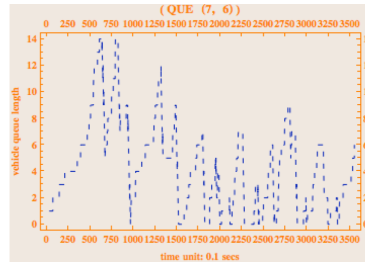

Figure 7. Queue $(7,6)$ - $(d_3, \{\mu^i\}_{i=1}^4)$ - MP control

Figure 8 illustrates the evolution of queue $q_{(7,6)}$ *within a waste of green time whenever phase* $(2,3)$ *is restrained and the corresponding actuation period is not reallocated to other movements.*

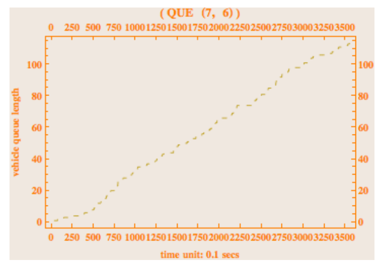

Figure 8. Queue $(7,6)$ - $(d_3, \{\mu^i\}_{i=1}^4)$ - MP control

IV. Future Work

Current studies focus on control aspects for saturated networks and the imposed restrictions by a limited queue size. The impact of the traffic density and network topology (shared lanes) to the saturation flow is being examined. Furthermore, the perspective of additional designs of adaptive controllers providing stabilising network performance are under conception. Implementations utilising real data are currently in process.

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