Fundamental Diagram Calibration: A Stochastic Approach to Linear Fitting

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Abstract

A new model is proposed for the relationship between the flow and density of traffic, otherwise known as the fundamental diagram, which captures both the nominal and the variations of flow. A statistical learning methodology is proposed for characterizing and identifying key parameters of the fundamental diagram that describes the dependence of traffic flow (or speed) on traffic density in a roadway section, based on traffic data obtained from a vehicle detection station. The proposed fundamental diagram characterization not only provides the expected value of flow (or speed) given a density measurement, but also a random probability distribution of the flow (or speed) given the density measurement. The former can be used to conduct deterministic traffic flow simulations, while the latter can be used to conduct statistical flow simulation studies, by using first order traffic flow models such as the cell transmission model.

Motivation

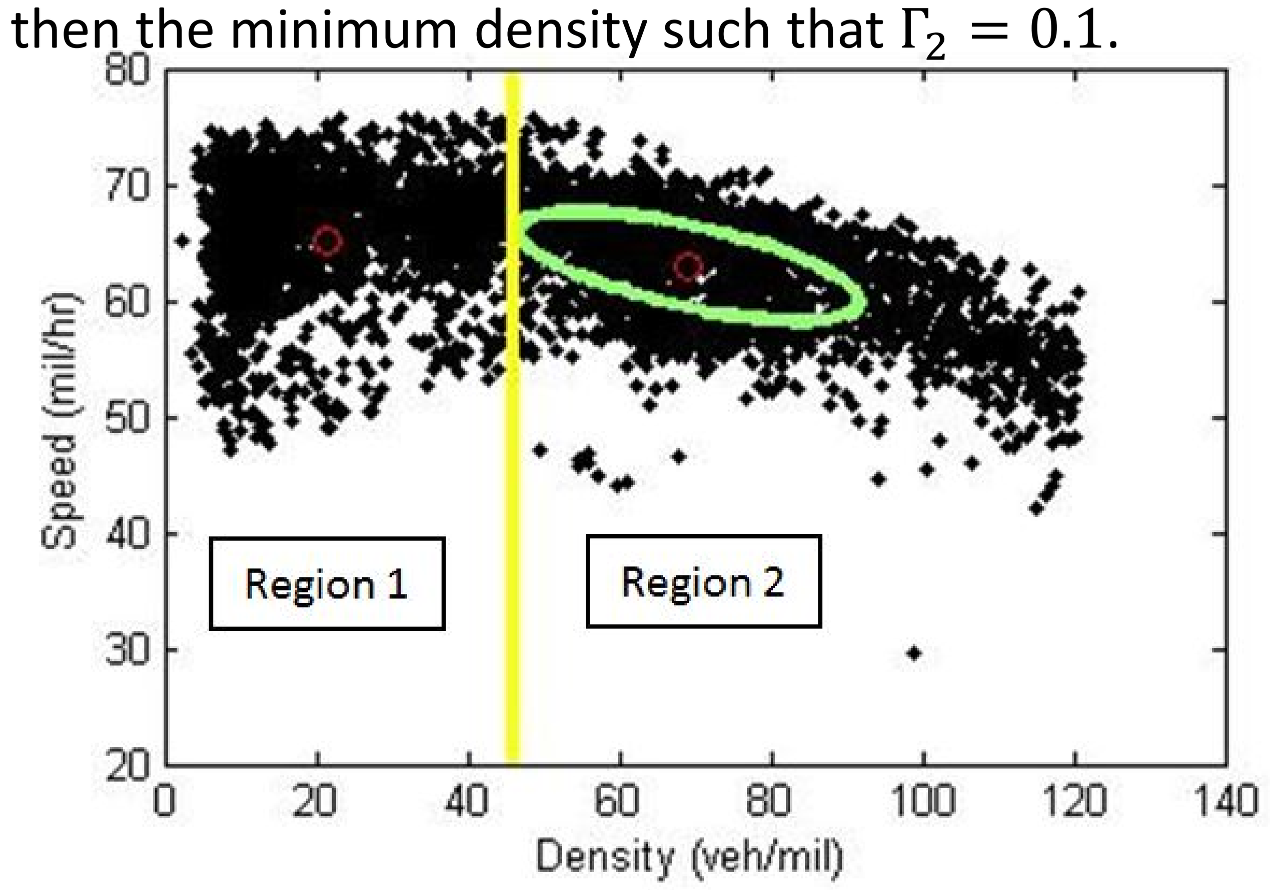
With detector technology currently available for measuring traffic flow and density, large sets of data can be collected over a period of days or months that show trends. However, to use this information, the data must be compressed into a set of small, reasonable parameters that capture both the trend and the variation of the data. Our paper works to define and determine those parameters.

Transition Density

The parameter indicates the density above which there is small, but noticeable, general reduction of speed, likely sourced from interactions between cars and slower vehicles. This value is determined by assuming two probability distributions, Γ_1 and Γ_2, among the data, x_n, of the free-flow regime (that which is below an assumed critical density) and using the Expectation-Maximization (EM) algorithm to determine the distribution parameters:

$$\tau_n^i = \frac{\pi_i \Gamma_i(x_n|\mu_i, \Sigma_i)}{\sum_j \pi_j \Gamma_j(x_n|\mu_j, \Sigma_j)}$$

where π_i, μ_i, and Σ_i are values determined as a function of τ_n^i and the data values. This is applied sequentially until convergence. The transition density is then the minimum density such that $\Gamma_2 = 0.1$.

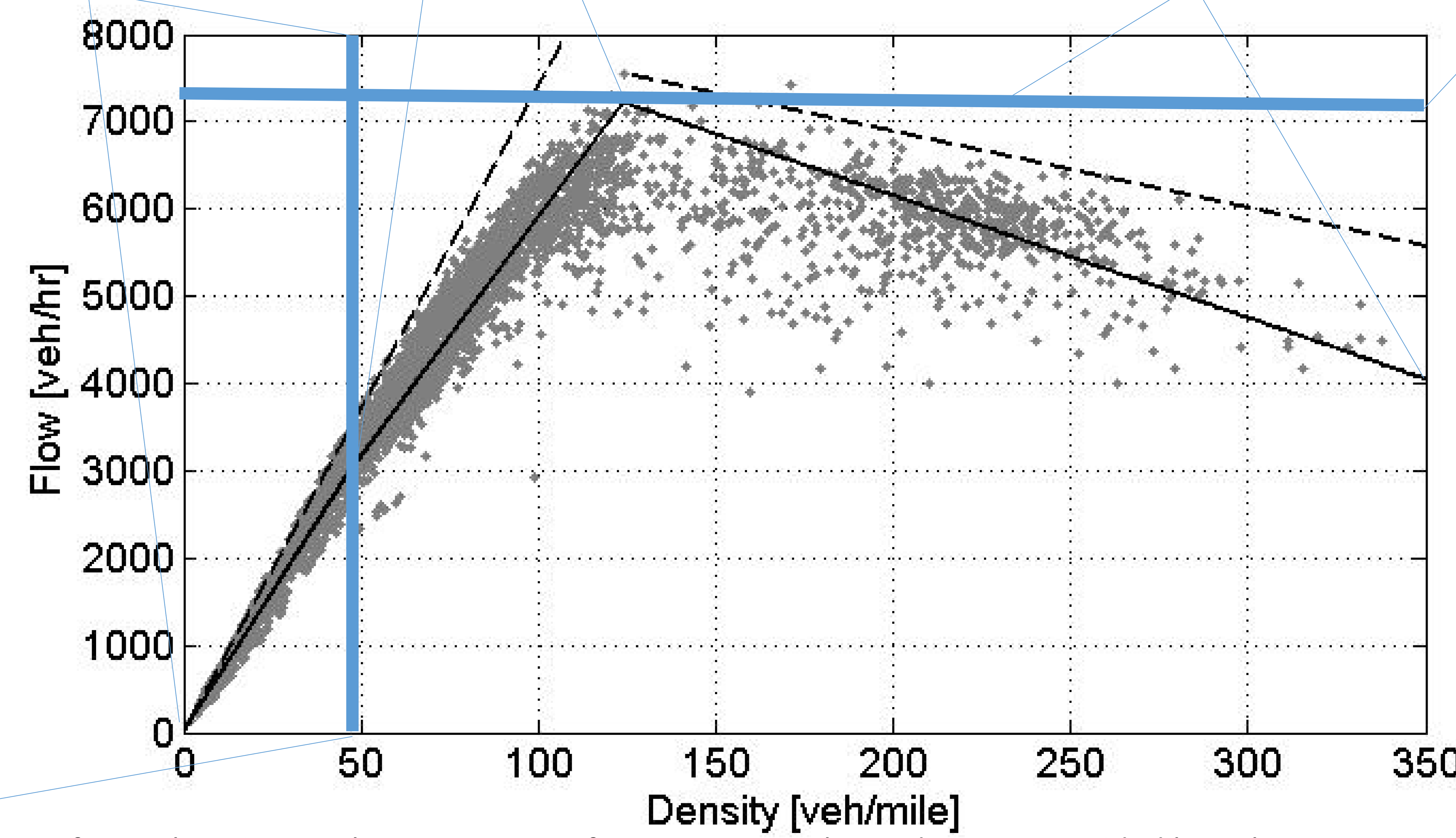


Generalized Linear Model

Given three regions of density have been defined by the transition density and the critical density, we seek to generate affine, continuous values that will convey information for the purposes of a fundamental diagram. It is possible to use linearization, but in considering the fundamental diagram as a function of flow from density, we also want to convey the conditional probability of flow given density, $P(f|\rho)$. If an approximation is made that this is an exponential distribution below an upper bound, we need only define the upper bound (in this case as the maximum value of data values/slopes) and find the lines such that every point is the expected value of flow given density at this value. Considering the Generalized Linear Model conceived above, an iterative algorithm can be used to determine the slopes of these lines:

$$\tilde{v}_{t+1} = \tilde{v}_t + \gamma(\tilde{f}_n - \tilde{v}_t\tilde{\rho}_n)(\tilde{v}_t\tilde{\rho}_n)^{-2}\tilde{\rho}_n$$

where γ is a step-size and $\tilde{v}$, $\tilde{f}$, $\tilde{\rho}$ are normalized values of speed, flow and density, respectively, for ordered data selection n and time step t of this iteration.



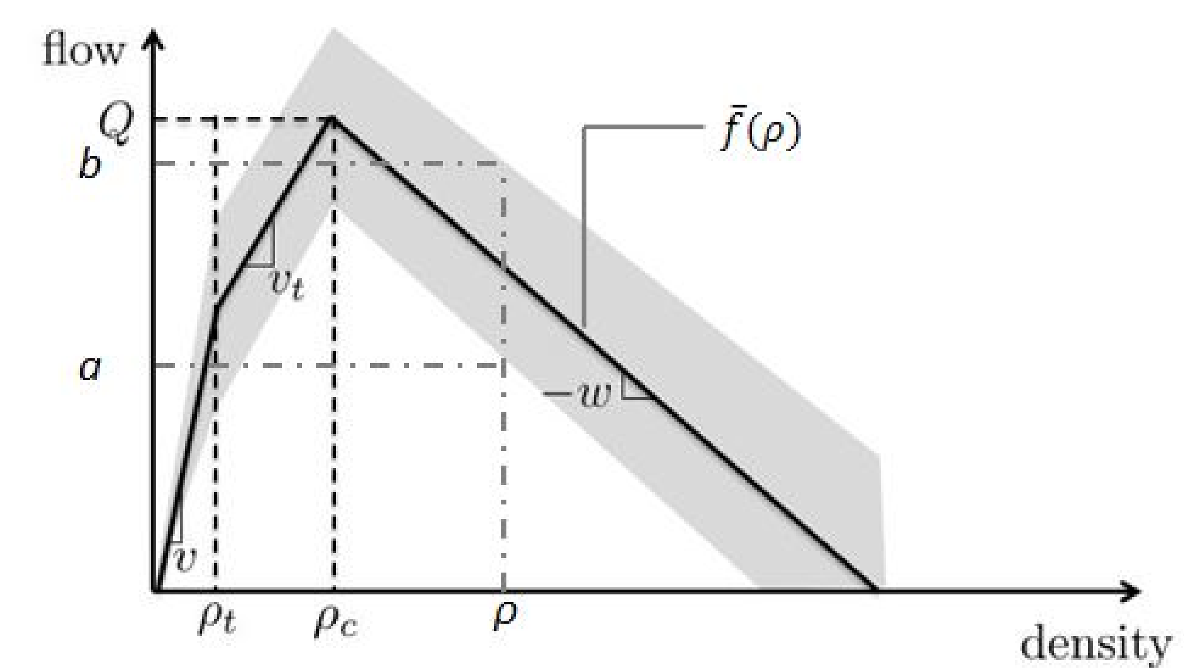
Data from a location on the Interstate 80 freeway near Richmond, CA, as provided by Caltrans Performance Measurement Systems (PeMS) with the determined relationship of flow and density at this location superimposed

Capacity

Capacity is the maximum amount of flow a location can allow at any time. We use capacity to redefine the critical density to maintain continuity. However, defining the maximum flow in the data as the capacity leads to difficulties in generating the mean value of the congested region. It is thereby appropriate to lower the capacity by some amount a decided by the user.

Conclusion

Using the concepts discussed here, a new flow-density relationship is available for use in traffic modeling and forecasting. The model can be used both deterministically or probabilistically. Further investigation can be made in the application of this model in real simulation contexts.



The new fundamental diagram. For a given density ρ, the value of flow will be between a and b.