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Optimization-Based Queue Estimation on an Arterial Traffic Link with Measurement Uncertainties

Leah A. Anderson, Edward S. Canepa, Roberto Horowitz, Christian G. Claudel, Alexandre M. Bayen

We present a queue estimation procedure that can be used to integrate measurements from typical count or occupancy sensors into an explicit physical model of arterial link state.

Background

$x \in [\xi, \chi]$	spatial location	*meters*
$t \in [0, t_{\max}]$	time	*seconds*
$\rho(t,x)$	lane density	*vehicles per meter (per lane)*
$f(t,x)$	lane flow	*vehicles per second (per lane)*
v	freeflow velocity	*meters per second*
w	queue dissipation speed	*meters per second*
ρ_c	critical density	*vehicles per meter*

Arterial traffic flow dynamics can be described by the **Lighthill-Whitham-Richards** (LWR) partial differential equation:

$$\frac{\partial \rho(t,x)}{\partial t} + \frac{\partial \psi(\rho(t,x))}{\partial x} = 0$$

where

$$\psi(\rho(t,x)) = \begin{cases} v\rho & \text{if } \rho \le \rho_c \\ w(\rho - \rho_c) & \text{otherwise} \end{cases}$$

Consider a function $\mathbf{M}(t,x)$ defined such that

$$\frac{\partial \mathbf{M}(t,x)}{\partial x} = -\rho(t,x) \quad \text{and} \quad \frac{\partial \mathbf{M}(t,x)}{\partial t} = f(t,x) = \psi(\rho(t,x))$$

Via integration of $\rho(t,x)$, the LWR equation can be rewritten as a Hamilton-Jacobi PDE in terms of the **Moskowitz** or **"cumulative number of vehicles"** function $\mathbf{M}(t,x)$, with Hamiltonian $\psi(\cdot)$:

$$\frac{\partial \mathbf{M}(t,x)}{\partial t} + \psi\left(-\frac{\partial \mathbf{M}(t,x)}{\partial x}\right) = 0$$

We find an explicit expression for $\mathbf{M}(t,x)$ using a class of weak solutions to HJ-PDEs known as the **Barron-Jensen/Frankowska** (B-J/F) solutions.

Lax-Hopf formula:

For a value (initial or boundary) condition $\mathbf{c}_j(\cdot,\cdot)$,

$$\mathbf{M}_{\mathbf{c}}(t,x) = \inf_{(u,T) \in \text{Dom}(\varphi^*) \times \mathbb{R}^+} \left(\mathbf{c}(t-T, x+Tu) + T\varphi^*(u)\right) \qquad (1)$$

where $\varphi^*(\cdot)$ is the *Legendre-Fenchel transform* of Hamiltonian $\psi(\cdot)$:

$$\varphi^*(u) := \sup_{p \in \text{Dom}(\psi)} [p \cdot u + \psi(p)] \qquad (2)$$

Because this is a weak solution, it does not guarantee that $\forall (t,x) \in \text{Dom}(\mathbf{c})$, $\mathbf{M}_{\mathbf{c}}(t,x) = \mathbf{c}(t,x)$.

Inf-morphism property:

Let $\mathbf{c}(\cdot,\cdot)$ be a minimum of a finite number of lower semicontinuous functions,

$$\forall (t,x) \in [0, t_{\max}] \times [\xi, \chi], \; \mathbf{c}(t,x) := \min_{j \in J} \mathbf{c}_j(t,x) \qquad (3)$$

Then $\mathbf{M}_{\mathbf{c}}$ can be decomposed as

$$\forall (t,x) \in [0, t_{\max}] \times [\xi, \chi], \; \mathbf{M}_{\mathbf{c}}(t,x) = \min_{j \in J} \mathbf{M}_{\mathbf{c}_j}(t,x) \qquad (4)$$

Methodology

$$y := (\rho(1), \ldots, \rho(k_{\max}), f_{\text{in}}(1), \ldots, f_{\text{in}}(n_{\max}), f_{\text{out}}(1), \ldots, f_{\text{out}}(n_{\max}))$$

We wish to determine a feasible y which can be used to reconstruct link state $\mathbf{M}(t,x)$ which satisfies both *model dynamics* and *measured data* for all t, x.

- **use the inf-morphism property to define linear *model constraints* on y;**
- **encode available measurements as additional linear *data constraints*;**
- **formulate an optimization problem on y to determine the most realistic feasible traffic state based on the set of available data constraints;**

Minimize: $g(y)$

subject to: $\begin{cases} A_{\text{model}}\, y \le b_{\text{model}} \\ C_{\text{data}}\, y \le d_{\text{data}} \end{cases}$

- **solve the relevant linear programs using a MATLAB-based optimization software package;**
- **use a separate MATLAB toolbox to generate the desired B-J/F solutions to the Moskowitz HJ-PDE, available at** `http://traffic.berkeley.edu/project/downloads/lwrsolver`.

Results

We ran our estimation algorithm using samples of high-resolution vehicle trajectory data from NGSIM's Lankershim Blvd dataset, `http://ngsim-community.org/`.

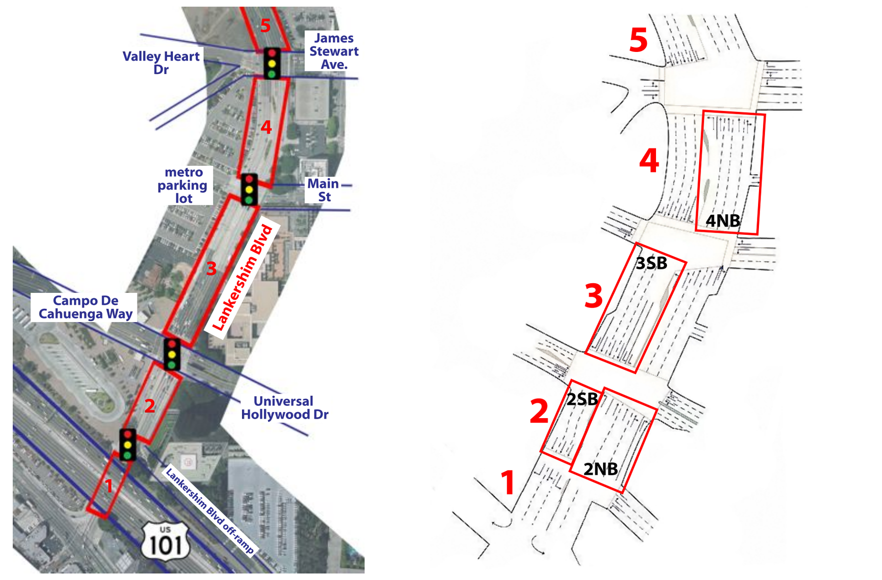
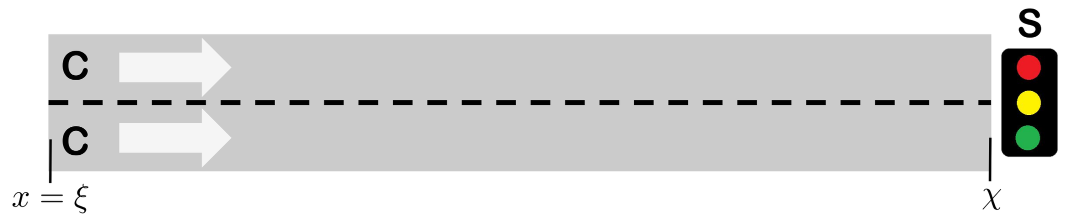


Figure 1: Lankershim Blvd data collection site

Scenario I: Vehicle count measurements at the upstream link boundary are aggregated into flow estimates $\bar{f}^k(T,\xi)$ for a fixed time step T. These measured flows have known error percentage $\bar{e_f}$. Known signal timings provide partial information about link outflow.



Relevant Data Constraints:

- $f_{\text{out}}(t_{\text{red}}) = 0$
- $(1-\bar{e_f})\bar{f}^k(t_k,\xi) \le f_{\text{in}}(k) \le (1+\bar{e_f})\bar{f}^k(t_k,\xi) \quad \forall\, t_k \in [k \cdot T, (k+1) \cdot T]$

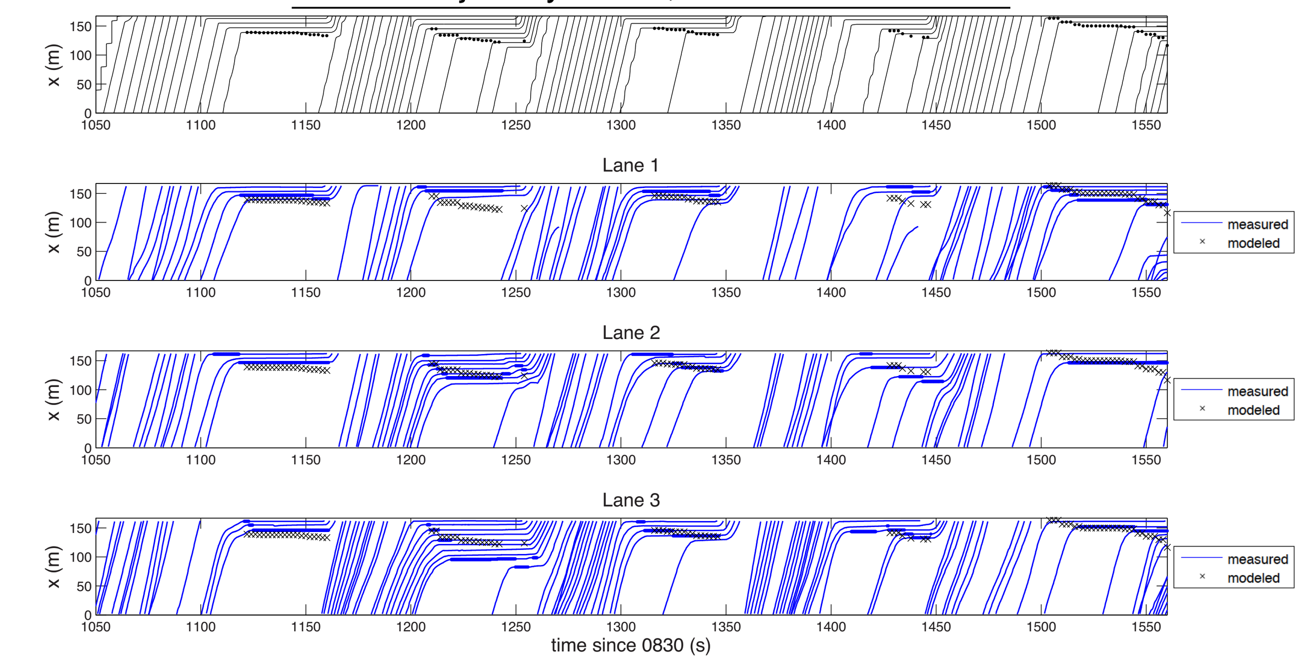
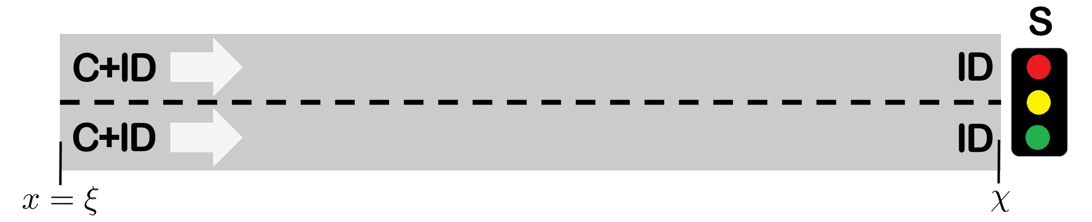


Figure 2: Model performance on through lanes of Link 3 SB. Inflow was reduced by 36% to account for turning vehicles. Modeled queues are represented in bold lines on all plots.

Scenario II: Flow measurements as in Scenario I are given. Additionally, re-identification sensors placed at the upstream and downstream ends of the link provide point-to-point travel times $\bar{t}$ with maximum error $\bar{e}_t$, corresponding to exit time stamps $\bar{t_f}$ for 5-15% of the vehicles traveling across the link.



Relevant Data Constraints:

- $f_{\text{out}}(t_{\text{red}}) = 0$
- $(1-\bar{e_f})\hat{f}^k(t_k,\xi) \le f_{\text{in}}(k) \le (1+\bar{e_f})\bar{f}^k(t_k,\xi) \quad \forall\, t_k \in [k \cdot T, (k+1) \cdot T]$
- $M(\bar{t_f} - \bar{t} - \bar{e}_t, \xi) \le M(t_f, \chi) \le M(\bar{t_f} - \bar{t} + \bar{e}_t, \xi)$ for $\bar{t}$, $\bar{t_f}$ sampled from 5-15% of exiting vehicles

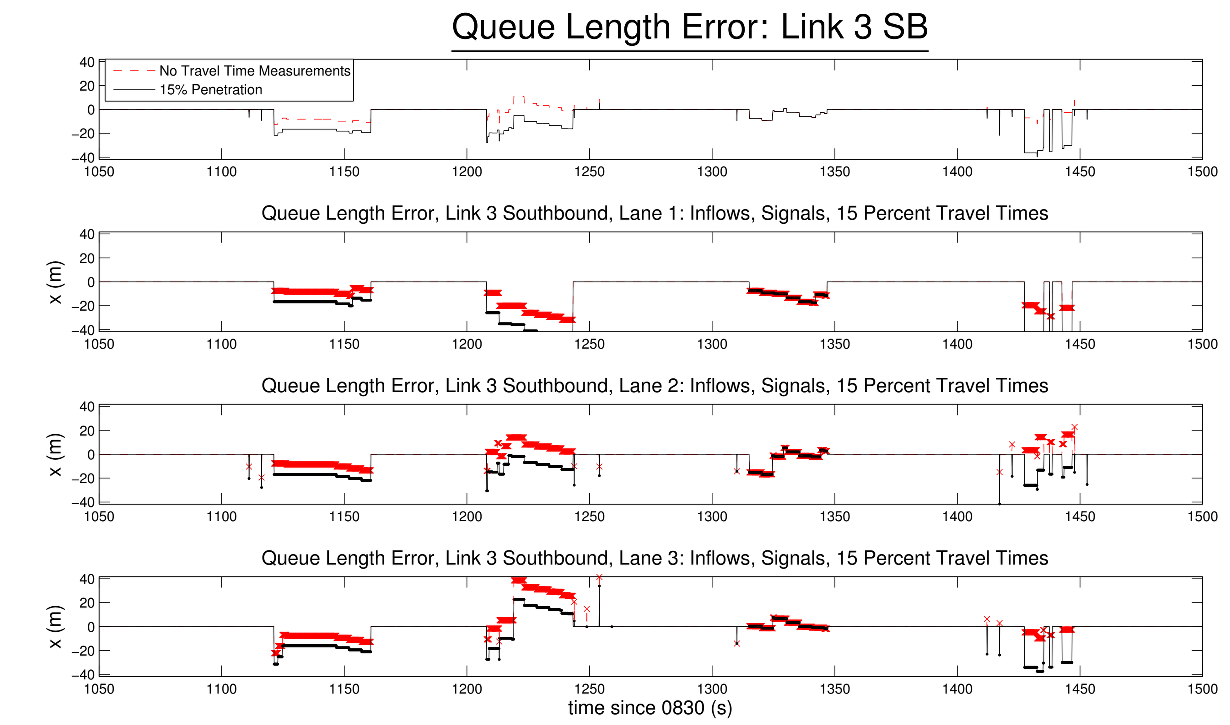


Figure 3: Travel time samples decreased the error for some lanes (relative to Scenario I) as they "tuned" the output to those lanes for specific queue cycles. However, the estimates were often made worse for the other lanes, causing increased error in the average length estimates for the affected cycles.

Mean Absolute Error in Queue Length Estimates:

Link	Scenario 1	Scenario 2 (15%)
2 SB	9.88 m	9.88 m
2 NB	14.73 m	19.30 m* (w/ 5%)
3 SB	13.69 m	15.53 m
4 NB	11.67 m	11.67 m

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